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HIGHER EDUCATION

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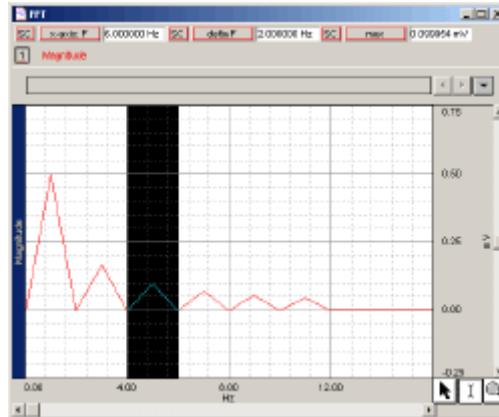info@biopac.com | www.biopac.com

BSL *PRO* Lesson H33:   FFT Fast Fourier Transform

This *PRO* Lesson details hardware and software setup of the BSL *PRO* software to examine the Fast Fourier Transform. All data collection and analysis is done via the BIOPAC MP35 or MP30 data acquisition unit and the BSL *PRO* software.

Objectives

This *PRO* Lesson will examine the Fast Fourier Transformation (FFT) as follows:

1. Build up a "square wave" composite waveform from sine components.
2. Use the FFT to analyze the "square wave" composite waveform.

Abstract

What is an FFT?

The **Fast Fourier Transformation (FFT)** is an algorithm that produces a description of time series data in terms of its frequency components. This is related to the *frequency spectrum* or *spectral analysis*.
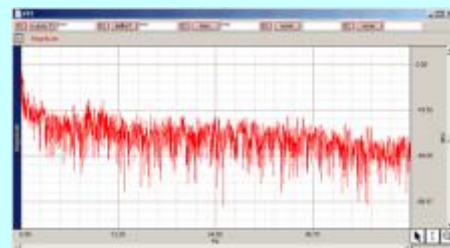
- **FFT**: Fast (assumes data is a power of 2 samples long); Time domain □♦ Frequency domain
- **IFFT**: Inverse; Frequency domain □♦ Time domain
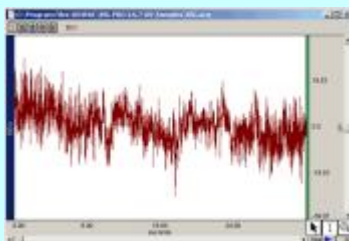
Time-based graph (amplitude/time)

FFT Parameters Dialog

FFT graph (magnitude/frequency)

The output from an FFT appears in a graph window with magnitude (vertical axis) plotted against various frequencies (horizontal axis). The range of frequencies plotted is from 0 Hz to 1/2 the sampling frequency. Thus, if data were collected at 200 samples per second, the BSL *PRO* software would plot the frequency components from 0 Hz to 100 Hz. Magnitude components for given frequencies appear as positive (upward-pointing) peaks.

A discrete Fourier transform results in a series of complex valued coefficients that are related to frequency. They are interpreted as the zero (DC) value, positive frequencies up to 1/2 the sampling rate, and negative frequencies up to 1/2 the sampling rate. When a real valued source signal is used in the amplitude/time domain, the magnitude of the negative frequency component of the Fourier transform results is equal to the positive frequency component. Because of this symmetry, BSL *PRO* displays only the positive frequency components, from which the negative frequency components can be inferred.

The "Linear" option displays an amplitude that is half of that needed to reconstruct the signal with a range of function generators*. To reconstruct a signal from additive sine waves, both the negative and the positive frequency components are required. Since it's not physically possible to generate a negative frequency signal, it's often reassuring to view the positive frequency components after doubling their respective amplitudes.

* This is why, in this lesson, the FFT decomposition shows the magnitude of the fundamental sine wave as 0.5 V when in fact the original fundamental sine wave associated with the composite "square wave" had a magnitude of 1 V.

Why use an FFT?

Fourier analysis can yield important information about the frequency components in a data set, and can be useful in making determinations regarding:

1. Interference resulting from periodic sources.
2. Signal power distribution versus frequency.
3. Choosing the appropriate filter to extract specific frequency ranges to optimize the desired signal to noise ratio.

What limitations does the FFT have?

1. The BSL *PRO* software cannot perform an FFT in real time (during acquisition). However, it is possible to emulate online spectral analysis using several online filters with the rectification and smoothing options applied to the filter outputs and then viewed via the Input Values Window.

2. The FFT algorithm assumes that the input time series data is an infinitely repeating periodic signal with the endpoints wrapping around. Thus, to the extent that the amplitude of the first point differs from the last point, the resulting frequency spectrum is likely to be distorted as result of this start point to endpoint discontinuity. This can be overcome by "windowing" the data during the transformation. *This lesson does not use windowing.*

3. The BPM and Freq measurements are not available with the FFT function.

FFT Terminology

From CHAMBERS DICTIONARY OF SCIENCE AND TECHNOLOGY (©Chambers Harrap Publishers Ltd. 1999):

|	
Joseph Fourier	Baron Jean Baptiste Joseph Fourier, 1768-1830. French mathematician and physicist who formulated a method for analyzing periodic functions and studied the conduction of heat.
Fourier analysis	The determination of the harmonic components of a complex waveform (i.e. the terms of a Fourier series that represents the waveform) either mathematically or by a wave-analyzer device.
Fourier principle	The principle that all repeating waveforms can be resolved in sine wave components consisting of a fundamental and a series of harmonics at multiples of this frequency. It can be extended to prove that non-repeating waveforms occupy a continuous frequency spectrum. **F(t) = $a_1 sinw_1$ + $a_2 sinw_2$ + $a_3 sinw_3$ +…**
Fourier transform	A mathematical relation between the energy in a transient and that in a continuous energy spectrum of adjacent component frequencies.

FFT in detail

The FFT is an orthogonal linear transformation, which changes the view from a function of time to a function of frequency. Since orthogonal planes are at 90 degrees to each other, any view from one plane has no bearing on the other plane(s). Fully orthogonal views are at right angles to each other, with no intersection or sense of data in the other planes. The FFT gives you a different window onto the data. The nuances of each domain are only clear in each view—all views are useful. When you look at something in terms of height, you have no perspective about length. The FFT is like an orthogonal view* port, with no aspect of any other view, such as the time series view. The FFT provides a view we aren't used to seeing. In the same way that you can view length or width but need both for the full perspective, the FFT shifts the view from the time domain to the frequency domain.
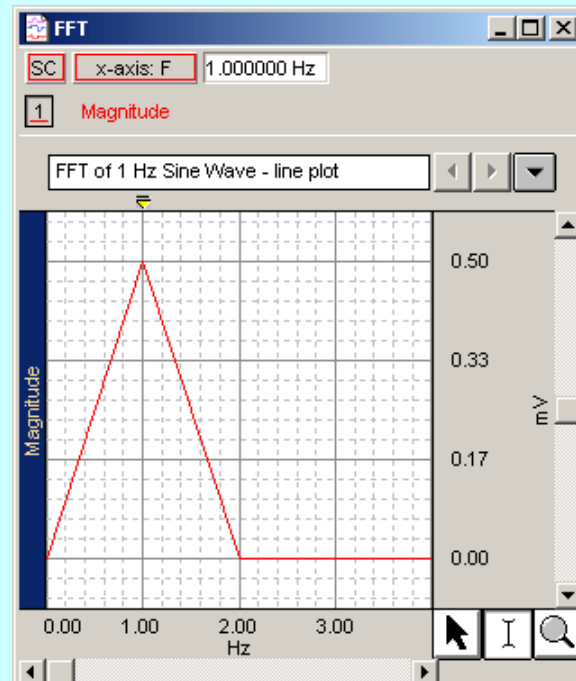
* Use of the term "view" is metaphorical; you will not visually perceive the FFT waveform—it is in a different dimension.

1 Hz Sine Wave
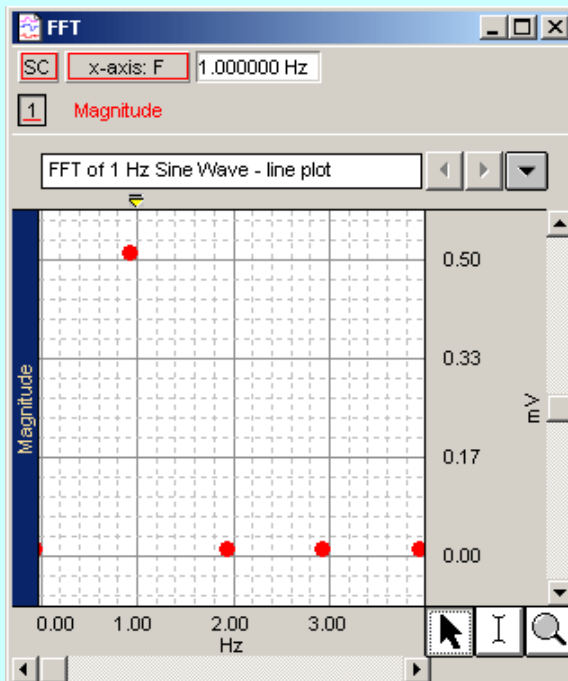
Dot Plot - FFT of 1 Hz Sine Wave
*Accurate data (series of frequencies)*

Line Plot - FFT of 1 Hz Sine Wave
Used to ease visual perception

The FFT is a series of frequencies. The dot plot view accurately shows an FFT, whereas the line plot view assumes data that does not exist in order to connect the data points and ease visual perception.

Analogy

Tie a rope to a doorjamb and shake it up and down you will generate a visual representation of a time-series sine wave, assuming the length of the rope represents time and the height of the rope swing is the amplitude of the sine wave. If you turn the rope in a circle at the same rate, you will generate a phasor representation of the sine wave, where the radius of the circle is the amplitude of the wave and the spinning rate of the rope is the frequency of the sine wave.

An infinite series of sine waves can be combined to create a square wave. If a finite series (i.e., three) of sine waves is arithmetically combined to create a "square wave" composite, the composite will only partially mimic a perfect square wave.
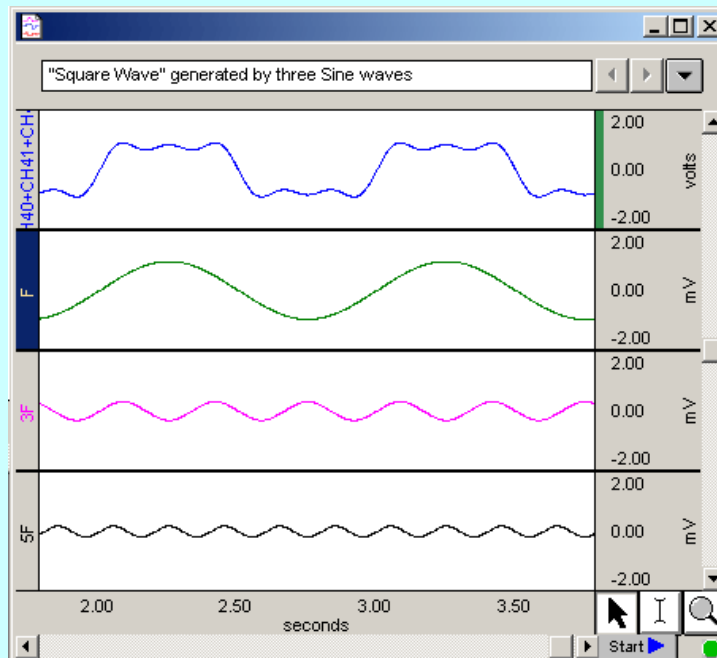
"Square Wave" generated from Expression: F + 3F + 5F
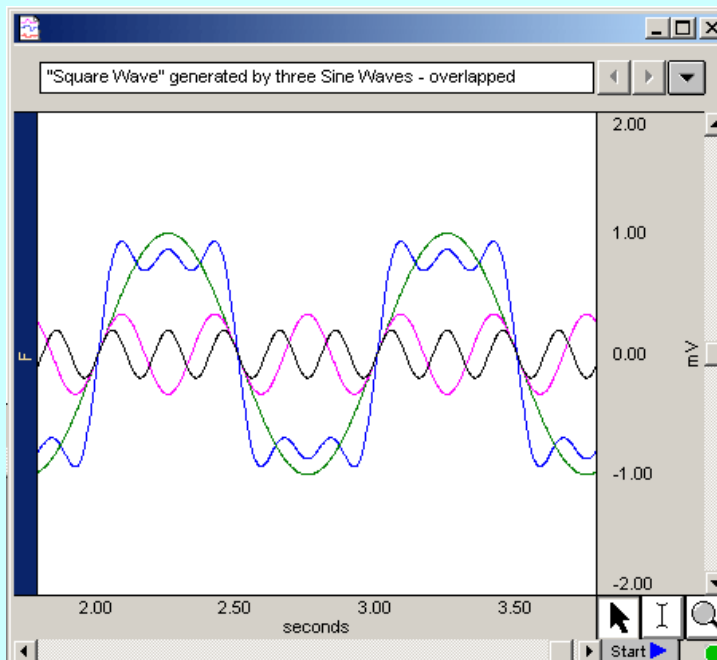
"Square Wave" Composite (blue) overlapped with sine wave components

Suggested sites for additional study:

- An Introduction to Fourier Theory *by Forrest Hoffman*
  http://lanoswww.epfl.ch/studinfo/courses/cours_dynsys/extras/Hoffman(1997)_An_Introduction_to_Fourier_Theory.pdf
- Introduction to Fourier Theory http://www.fftw.org/links.html
- Fourier Transform from MathWorld http://www.fftw.org/links.html

Equipment

- Computer running Windows XP or Mac OS X
- Biopac Student Lab *PRO* software
- BIOPAC Data Acquisition Unit (MP35/MP30)

Setup

Hardware

1. Make sure that the MP3X unit is connected to the computer.
2. Turn on the computer and the MP3X unit.

Software

1. Launch the **BSL *PRO*** software.
2. Open the *FFT* template file by choosing **File menu > Open > choose Files of type: GraphTemplate (*GTL) > File Name: h33.gtl**.

   - The template should be set for the following channels:

Channel	Displays
CH 1	not plotted
C1 (40)	1 F (frequency)
C2 (41)	3 F
C3 (42)	5 F
C4 (43)	7 F
C5 (43)	9 F
C6 (45)	11 F
C7 (46)	SUM (C1:C7)

Calibration

- No calibration required.

Recording

1. Click **Start** in the software to begin acquiring data.

Analysis

Perform an FFT on the "Square Wave" composite from this recording to identify the contributing frequencies and corresponding amplitudes that were used to create the "Square Wave" composite.

Segment 1- Sine Wave Components

1. Resample to 512 samples: **Transform** > **Resample** > **512**).
2. *Select about 1 second of data to capture **512 or less samples**.*
   - Select the data so the low (or high) going portion of the composite "Square Wave" (SUM) is centered in the selection area.
   - Select the number of samples to match the period.
     For example: 1 cycle = 512 samples = 512 point FFT
   - **It's important** to select as close to 512 as possible without exceeding 512 since windowing is not used in this lesson—see below:
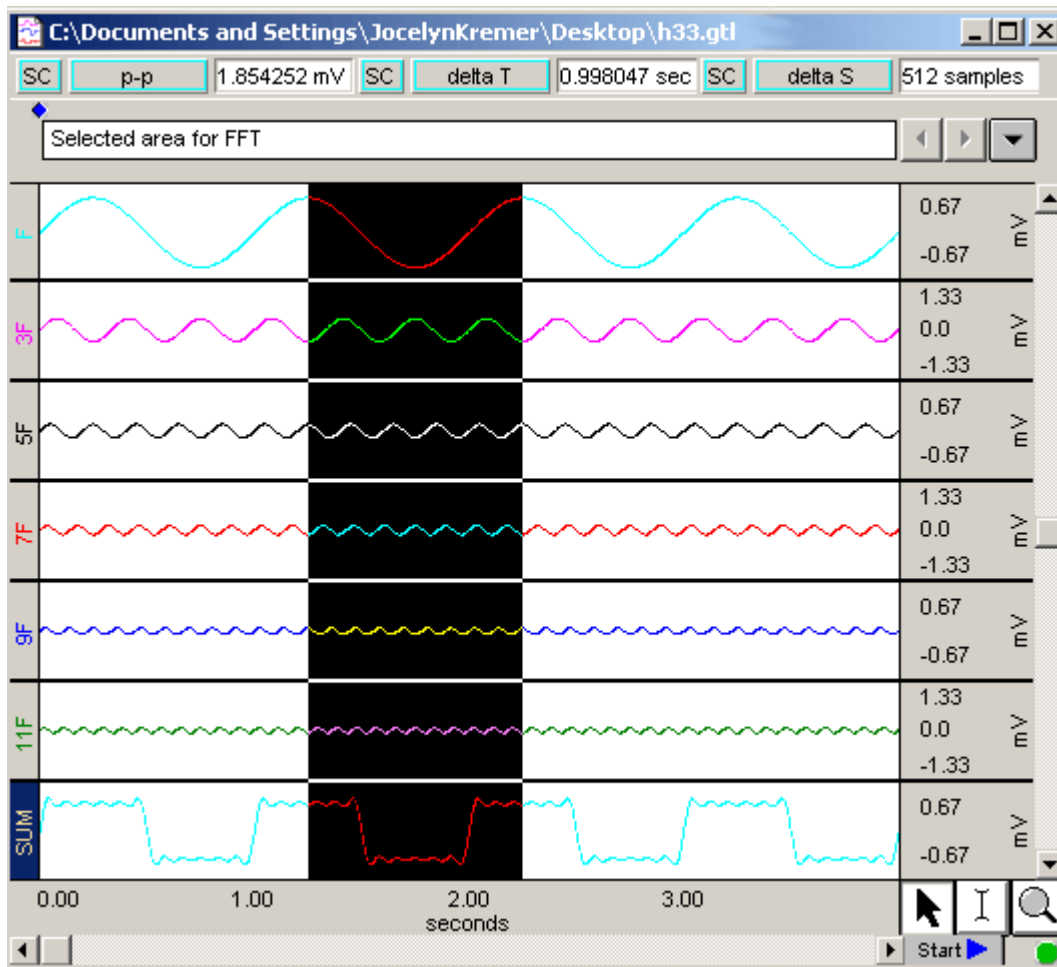

Selected area with 512 samples for FFT

3. Select the SUM channel and perform an FFT with Magnitude, Linear, and no Window.
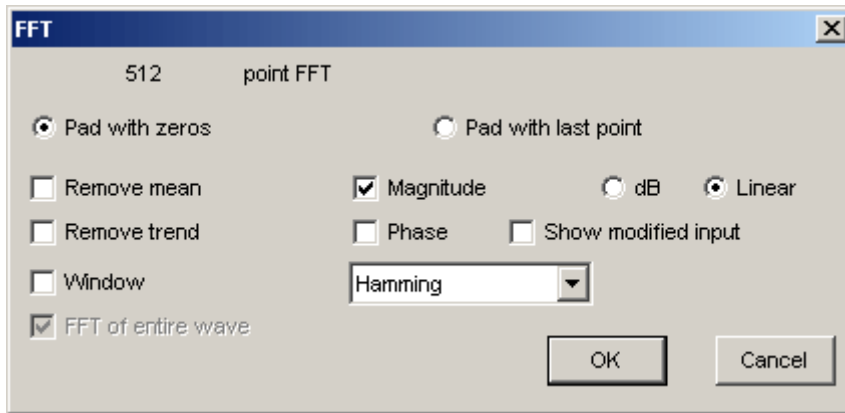
4. *Note —* The FFT dialog contains other items not required for this lesson, but often useful:

    Remove mean—gets rid of DC value, which can create an initial spike if the waveform is not at zero.
    Remove Trend—calculates slope of start and end. Helps minimize transition from the wrap-round effect.

5. Autoscale the display (**Display** > **Autoscale** waveforms).
6. Measure the amplitude and frequency of each spike in the FFT to validate the composite wave.
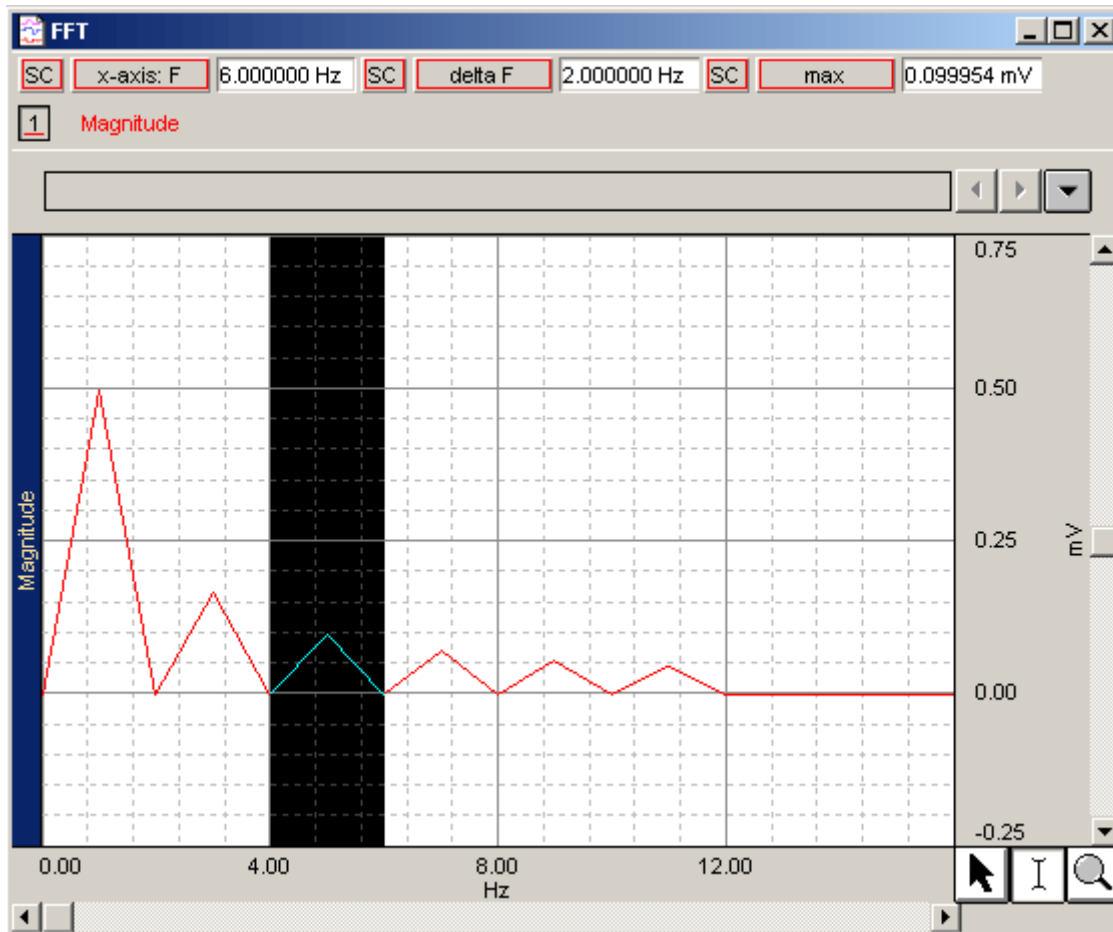
Segment 2- Square Wave Comparison

In this segment, you will add a square wave and then overlap with the sine wave components to analyze the components.

1. Select CH 1 "F" and then duplicate it (**Edit** > **Duplicate Waveform**).
2. Select CH 2 "F" (the duplicate waveform).
3. Select all (**Edit** > **Select all**).
4. Perform a Threshold function at "0" (**Transform** > **Math Functions** > **Threshold** > "**0**").
5. Apply an expression to make the square wave ±1 V (**Transform** > **Expression** > (CH2*2)-1).
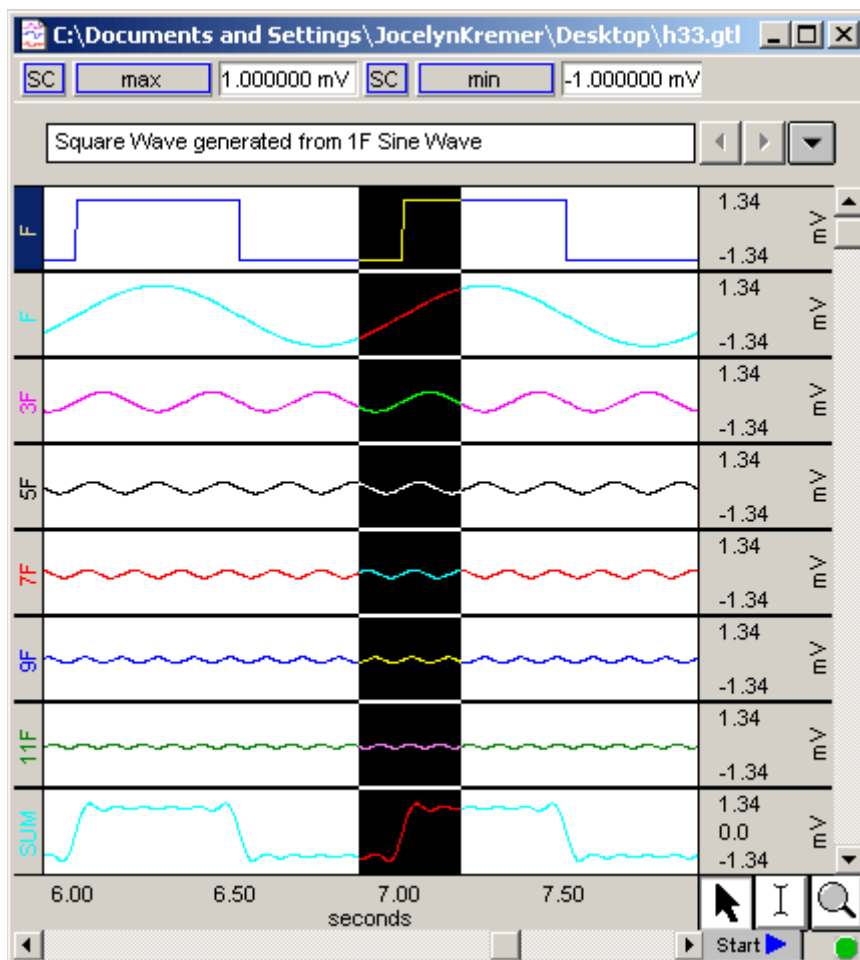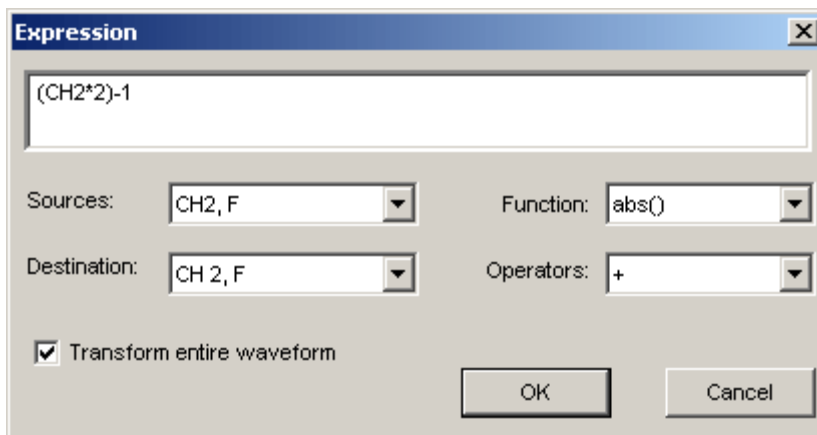
6. Activate the Scope mode (**Display** > **Show** > **Scope** mode or toolbar icon).
7. Overlap (**Display** > **Overlap**) the waveforms.
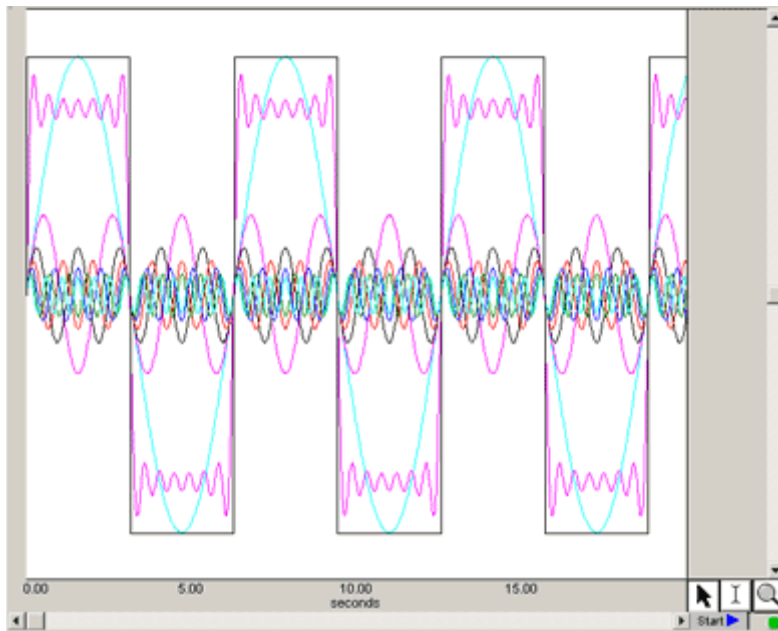
8. Adjust the display (zoom, scale, etc.) as desired to analyze the components.
   o Note that the SUM wave looks more like the square wave than any of the original sine waves. If you have an infinite series of frequencies, the SUM wave will be a perfect square wave.
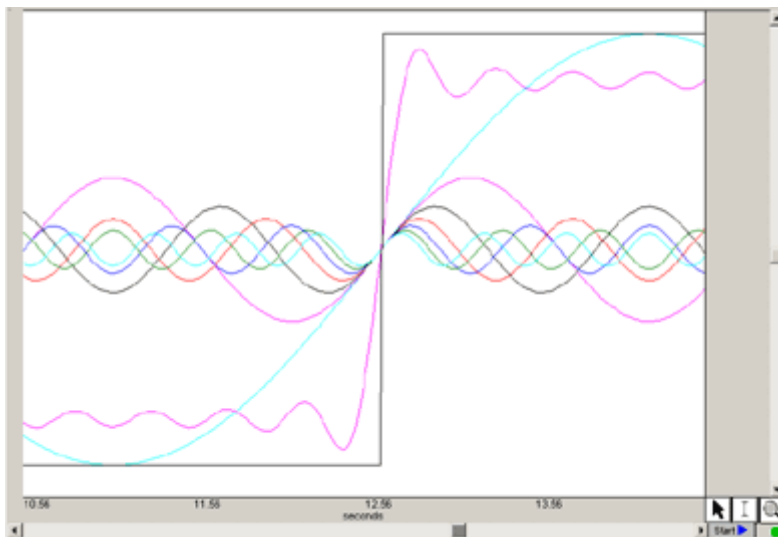
9. If desired, add or remove component waves to the SUM wave and note the effect.